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Inflation from N-Forms and its stability

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Abstract: We investigate whether inflation, either isotropic or anisotropic, may be supported by n-forms. Canonical field strengths and their duals are taken into account, and they are allowed to have a potential and also, when necessary for slow-roll, a nonminimal curvature coupling. New isotropic solutions are found for three-forms. It is also shown that some n-form actions are equivalent to f(R) gravity and scalar field models with possible nonminimal couplings. Anisotropic solutions are found for two-forms, generalising vector inflation. However, as the later also the two-form is unstable during inflation due to the nonminimal coupling to curvature. The stability of the isotropic background solutions supported by a triad of vectors is also analysed.

Keywords: Inflation, N-forms.

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1. Introduction

Inflation is an early era of accelerated expansion of the universe which was introduced to explain the homogeneity and isotropy of the universe at large scales [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Evolution of such a Friedmann-Robertson-Walker (FRW) universe can be described by a single scale factor. Whatever the matter contained there, its energy density behaves like a scalar at large scales, and it is convenient to consider inflation to be driven by a one or several scalar fields. However, at a more fundamental level there is no motivation to exclude the possibility of the energy source of the inflationary expansion having a nonscalar nature. In particular, higher spin bosonic fields could form condensates, and although perhaps surprising, slow roll inflation can be made possible even for massive vector fields, either time-like or space-like [11, 12]. A vector inflation was first proposed by Ford [13]. Even before, it was known that with the conformal coupling, the vector field equation of

motion is just like that of a scalar field, however the vector might appearing unstable due to its effective negative mass-squared during inflation [14, 15]. It is interesting to look for alternative scenarios to understand how generic such results are.

Such models have been overlooked since they generically induce an anisotropy. This picture has changed in the last couple of years. The recent detections of some unexpected features in the CMB temperature anisotropies have raised a lot of speculations about the need to reconsider some of the basic cosmological assumptions. A hemispherical asymmetry has been reported [16]. The angular correlation spectrum seems to be lacking power at the largest scales [17]. The alignment of the quadrupole and octupole (the so called Axis of Evil [18]) could also be seem as an extra-ordinary and unlikely result of statistically isotropic perturbations, even without taking into account that these multipoles happen also to be aligned to some extent with the dipole and with the equinox. The Axis of Evil does not show any correlation with the lack of angular power [19].

The significance of the anomalies has been debated extensively in the literature (see e.g. [20, 21, 22, 23]) with some reported effects more significant than others [24, 25, 26]. The difficulty in quantifying exactly the importance of any effect is due to how correctly the a posteriori probability of observing is estimated. It is clear however that the anomalies implying an overall anisotropy in the data are more significant than the lack of power in the CMB quadrupole. A natural explanation for the observed anomalies may be some form of a yet undetermined systematic or foreground signal which is not being taken into account properly in the data reduction producing the final maps [27, 28, 29, 30]. However, a conclusive explanation along these lines has not been put forward yet. It is therefore legitimate to ask whether the observed anomalies may be an indication of a departure from the standard cosmological model. CMB data strongly support the general theory of inflation, we focus here on departures from the standard picture which can however be reconciled with the inflationary framework.

With these considerations in mind, in this work we perform an initial step towards the study of a n-form -driven inflation. The basic questions we want to address are: do we know that the inflaton is a scalar? Is it isotropic? (By isotropic we will always mean spatially isotropic). We mostly consider models where the comoving field is dynamical, light and slowly rolling. Comoving field is the square of the form, and as a scalar it is the suitable degree of freedom to consider. By dynamical we mean that the field obeys second order equations of motion. By light we mean that the effective mass term appearing in the background equation is smaller than the Hubble rate. By slowly rolling we mean that the derivatives of the field are small compared to the Hubble rate. All these restrictions are not index necessary for a successful and viable inflationary scenario: indeed we show that some models could have only first order equations of motion and that a three-form can easily inflate the universe even if its comoving field is not slowly rolling. Our main aim is to find the *simplest* possible quadratic action that is compatible with inflationary solutions in each of the five classes of models we discuss (in four dimensions, one can have forms with five different index numbers 0, 1, 2, 3 or 4). During the final stages of this work, other works on form-driven inflation and the generation of gravitational waves within them have appeared [31, 32, 33, 34]. However, our starting action (2.4) is different from theirs which

includes nonminimal couplings with coefficients fixed in such a way that the equation of motions are always exactly of the scalar field field form. Thus there is only partial overlap.

The plan of the paper is as follows. In section 2 we review general properties of forms. First we motivate the basic action (2.4) which we write down in subsection 2.1, and whose dual we write down in subsection 2.2. In subsection 2.3 we consider the Stückelberg method of restoring the gauge-invariance of the models by adding a new field, which thus illuminates the field content of the theory. In section 3 we analyse each of the ten cases in turn. We review the vector inflation and show that its two-form generalisation is qualitatively similar in all respects. In general, both need nonminimal coupling in the slow-roll case, are not compatible with isotropy and seem to have instabilities. The latter is indicated also by an explicit computation. We also analyse the case that the background is made isotropic by considering several mutually orthogonal fields, the so called "triad" case. In contrast, the three-form does not need nonminimal coupling, is compatible with isotropy and stable. In general, the dynamics nonspatial forms is restricted due to the symmetries of the FRW and Bianchi I backgrounds. Still, one may construct viable models from them. We also find a reformulation of the nonlinear gravity theory (the $f(R)$ models) as a four-form action. All these results are concisely summarised in the Table 2 and in the concluding section 5.

2. Forms

String theories are generically inhabitated by forms and one could expect such to appear in effective low energy actions. As they often are accompanied by axions and "eat up" lower order forms, we know that consideration of massive forms can be well motivated. One possible manifestation of forms is antisymmetric gravity. Already Einstein attempted a geometric unification of General Relativity (GR) and electromagnetism by considering an asymmetric metric. Leading contribution from the antisymmetric part indeed includes a Maxwell type form, however the theory fails to predict the Lorentz force law correctly. Nonsymmetric generalisations of GR have continued to be of interest in theories and also phenomenology [35]. In particular, an antisymmetric tensor field is a crucial ingredient in some gravitational alternatives for dark matter [36]. On the other hand, this presents a natural way to generate propagating torsion. General actions for forms can break gauge invariance which easily could awake ghost or tachyon modes that otherwise remained unphysical gauge modes. Nonsymmetric theories may thus be stringently constrained, though at nonlinear level some instabilities could be avoided [37, 38]. However, some simple stable actions exist.

Motivated by these theoretical interests in unified theories at both classical generalisations of GR and in fundamental theories for quantum gravity, in the present study we consider the possibility to employ the stable types of the form field actions in cosmology where unknown fields are indeed needed, namely as energy sources for the present acceleration and inflation. The quantum generation of two-form fields during inflation has been considered [39]. Here we instead study form field driven inflation. Presence of n-forms could also explain the origin of four large dimensions, since due to inherent anisotropy of forms one might consider scenarios where only three spatial directions inflate while extra

dimensions were stabilised [40]. However, our focus here is on a possible residual anisotropy that could be (or even could have been as discussed in the introduction) observed in CMB, and in cosmological calculations (from the next section onwards) take a four dimensional action as our starting point.

To motivate the particular form of the action we consider, we begin by reviewing some general results about two-forms. In flat space, the most general quadratic, Lorentz- and parity invariant Lagrangian reads for a two-form A as

$$\mathcal{L}_f = -\frac{1}{4}a(\partial A)^2 - \frac{1}{2}b(\partial \cdot A)^2 - \frac{1}{4}m^2A^2, \tag{2.1}$$

where the dot product means contraction over the first indices. van Nieuwenhuizen has shown that unless $a(a+b) = 0$ the propagator contains nonlocality or a ghost [41]. This leaves two possibilities. If $a = -b$, one may show by partial integrations that the Lagrangian reduces to $\mathcal{L}_f = -aF^2/48 - m^2A^2/4$, where F is the Maxwell tensor formed from A, which in general is defined for a n-form as follows

$$F_{M_1 \ldots M_{n+1}}(A) = (n+1)\partial_{[M_1}A_{M_2 \ldots M_{n+1}]}\,, \tag{2.2}$$

where the square brackets indicate antisymmetrisation, for example $A_{[MN]} = (A_{MN} - A_{NM})/2$ and so on. The other possibility, that $a = 0$, corresponds to the dual theory, as will become clear in 2.2. Thus, in the flat space limit our theory ought to reduce to either a massive Maxwell i.e. Proca theory, or its dual. (One knows that this applies to the vector case as well since then, in addition to the Maxwell term, a gauge-fixing term which is got by $a = 0$ and actually reduces to a scalar theory, is the only consistent choice for a kinetic term in vacuum).

Apparently, in curved space more possibilities emerge. The form field might have couplings with the curvature tensors. The general quadratic couplings then include contributions from the three terms

$$\mathcal{L}_c = -\frac{1}{2}\sqrt{-g}\left(\xi RA^2 - cR_{MN}A^{NK}A_K{}^M - dR_{MNKL}A^{MN}A^{KL}\right)\,. \tag{2.3}$$

Note that for example couplings of the type RF^2, $R_{MN}F^{MK}F_K{}^N$ or $R_{MNKL}F^{MN}F^{KL}$ would result in higher-order derivative theory. Thus we leave them out, though it is well known such could be motivated by quantum corrections [42] (for recent higher order gravity-vector investigations see [43, 44]). However without unreasonable fine tunings, the stability of cosmology requires $c = d$, and to have always stable Schwarzchild solutions one must further set $d = 0$ [45]. Hence we are left with only a possible nonminimal coupling to the Ricci scalar R. Therefore our starting point will be a Maxwell action with a mass and a curvature coupling terms as free parameters. The mass can be promoted to a more general potential function $V(A^2)$ without complicating the analysis.

2.1 Equations of motion

Thus, we consider an n-form A in d dimensions with the following action

$$S = \int d^dx\sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2(n+1)!}F^2 - V(A^2) - \frac{1}{2n!}\xi A^2R\right)\,, \tag{2.4}$$

where $\kappa = 1/\sqrt{8\pi G_N}$ where G_N is the Newton's constant. The kinetic term is given by the square of the field strength,

$$F^2 = F_{M_{n+1}} F^{M_{n+1}}, \qquad F \equiv (n+1)\,[\partial A] \equiv \mathrm{d}A \tag{2.5}$$

In the cases where the indices are not written explicitly, the big square brackets mean antisymmetrisation. The last equality also defines the exterior derivative. Squaring means, here and elsewhere, contracting the indices in the same order. In the following, dotting means contracting the first index, and gradient means adding an index by differentiating. Note that the second line (2.5) is nothing but the definition (2.2) in the new more compact notation. We also use the abbreviation

$$M_n \equiv M_1 \dots M_n \,. \tag{2.6}$$

With these notations, the stress tensor found by metric variation can be written as

$$\begin{aligned} T_{MN} = & \frac{1}{n!} F_{MM_n} F_N{}^{M_n} + 2nV'(A^2) A_{MM_{n-1}} A_N{}^{M_{n-1}} - g_{MN}\left(\frac{1}{2(n+1)!}F^2 + V(A^2)\right) \\ & + \frac{\xi}{n!}\left[nRA_{MM_{n-1}}A_N{}^{M_{n-1}} + \left(G_{MN} - \nabla_M\nabla_N + g_{MN}\Box\right)A^2\right], \end{aligned} \tag{2.7}$$

where the second line is the contribution from nonminimal coupling. We note that the kinetic piece is traceless when $d = 2(n+1)$: the Maxwell field in $d = 4$ is conformal i.e. traceless. Furthermore, the potential piece is traceless if $V(x) \sim x^p$ where $p = d/(2n)$: a $(d/2-1)$-form in any d could be conformal with an interaction term $p = d/(d-2)$.

The equations of motion are

$$\nabla \cdot F = \left(n!2V' + \xi R\right) A, \tag{2.8}$$

implying thanks to antisymmetry

$$\nabla \cdot \left[\left(n!2V' + \xi R\right) A\right] = 0\,. \tag{2.9}$$

2.2 The dual theory

There are various theories of fundamental physics where duality transformation plays a role. Therefore it is motivated to consider also the dual actions for each n-form. If the forms are nonminimally coupled to gravity, the usual duality invariance does *not* hold at the level we consider. The Hodge duality transforms an n-form into a $(d-n)$-form. Component wise, the transformation rule is

$$(*A)_{M_{d-n}} = \frac{1}{n!}\epsilon_{N_n M_{d-n}} A^{N_n}. \tag{2.10}$$

It follows that $*(*A) = \mathrm{sgn}(g)(-1)^{(d-n)n}A$. The strength of a dual is not the same as the dual of the strength. The Hodge star operator should be applied to each form, namely to the strength appearing in the action (2.4), as it is an $(n+1)$-form. We get

$$(*F)_{M_{d-n-1}} = \frac{1}{(n+1)!}\epsilon_{N_{n+1}M_{d-n-1}} F^{N_{n+1}}. \tag{2.11}$$

From this follows, by using the identity (2.10) and the definition (2.5), that

$$(*F) = (-1)^n \nabla \cdot (*A). \tag{2.12}$$

Note also that by defining $\delta \equiv \mathrm{sgn}(g)(-1)^{dn+d+1}*\mathrm{d}*$, this can be recast in the more compact form $(*F) = (-1)^{n+1}\delta(*A)$. For a review of differential geometry and gauge theories see [46]. We find also $(*A)^2 = \mathrm{sgn}(g)(d-n)!A^2/(n!)$. Thus a spacelike field is transformed into a timelike and vice versa. The dual action for (2.4) is then written as

$$S = \int d^dx\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2(d-n-1)!}\left(\nabla\cdot(*A)\right)^2 - V\left((*A)^2\right) - \frac{1}{2(d-n)!}\xi(*A)^2R\right] \tag{2.13}$$

Thus the Maxwell-type kinetic term transforms to a square of a divergence. We see that indeed the dual has the unique form that is stable in flat space, got by setting $a=0$ in Eq.(2.1). The contribution to the stress tensor from this term constitutes the following two lines, the third is due to the potential, and the fourth vanishes if coupling to gravity is minimal:

$$\begin{aligned} *T_{MN} = & \frac{1}{(d-n-1)!}\Big[2\left(\nabla_{(M}(*A)_{N)M_{d-n-1}}\right)\left(\nabla_K(*A)^{KM_{d-n-1}}\right) \\ & + (d-n-1)\left(\nabla_L(*A)^L{}_{MM_{d-n-2}}\right)\left(\nabla_K(*A)^K{}_N{}^{M_{d-n-2}}\right) - \frac{1}{2}g_{MN}\left(\nabla\cdot(*A)\right)^2\Big] \\ & + 2(d-n)(*A)_{MM_{d-n-1}}(*A)_N{}^{M_{d-n-1}}V'((*A)^2) - g_{MN}V((*A)^2) \\ & + \frac{\xi}{(d-n)!}\left[(d-n)R(*A)_{MM_{d-n-1}}(*A)_N{}^{M_{d-n-1}} + \left(G_{MN} - \nabla_M\nabla_N + g_{MN}\Box\right)(*A)^2\right]. \end{aligned} \tag{2.14}$$

The potential and coupling terms are similar to those in (2.7). We note a massless dual is traceless if $n = d/2+1$, and a power-law potential leaves no trace if $p = d/(2(d-n))$. The equations of motion for the field are

$$[\nabla\nabla\cdot(*A)] = 2\left((d-n-1)!V' + \frac{1}{2(d-n)}\xi R\right)(*A). \tag{2.15}$$

Note that we can deduce only about the antisymmetric part of the variation since we are varying with respect to $(*A)$. One can show that the $(d-n)$-form with an action (2.13) can be equivalent to a canonical $(d-n-1)$-form $\phi_{M_{d-n-1}}$ by introducing Lagrange multipliers to fix $\phi = -\nabla\cdot(*A)$ The resulting action is

$$S = \int d^dx\sqrt{-g}\left[\frac{1}{2\kappa^2}R - V((*A)^2((\nabla\phi)^2)) - \frac{1}{2(d-n-1)!}\phi^2 - \frac{1}{2(d-n)!}\xi(*A)^2((\nabla\phi)^2)R\right], \tag{2.16}$$

where $(*A)^2$ is given by inverting

$$\left(\frac{\xi}{(d-n)}R + 2(d-n-1)!V'((*A)^2)\right)^2(*A)^2 = [\nabla\phi]^2. \tag{2.17}$$

If $V((*A)^2) = \frac{1}{2(d-n)!}m^2(*A)^2$ and we rescale $\phi \to \phi m$, we get

$$S = \frac{1}{2}\int d^dx\sqrt{-g}\left[\frac{1}{\kappa^2}R - \frac{1}{(d-n)!(\xi R/m^2+1)}[(d-n)\nabla\phi]^2 - \frac{m^2}{(d-n-1)!}\phi^2\right].$$

If $\xi = 0$, this is nothing but the canonic $(d-n-1)$ form since $[(d-n)\nabla\phi] = \mathrm{d}\phi$ is the strength of ϕ. In that case, the duality becomes trivially transparent in a more elegant notation [46]. Since the equation of motion for the dual can be written as

$$\mathrm{d}\delta(*A) + m^2(*A) = 0, \tag{2.18}$$

and if we choose $m\phi = \delta(*A)$, it is clear that the two terms that appear in the quadratic action can be expressed in terms of ϕ as

$$(*F) \wedge (**F) = m^2\phi \wedge (*\phi), \tag{2.19}$$

$$m^2(*A) \wedge (**A) = \mathrm{d}\phi \wedge (*\mathrm{d}\phi), \tag{2.20}$$

where by $(*F)$ we mean of course $(-1)^{n+1}\delta(*A)$. Let us make some remarks about different cases. We note that to get a more general potential for the field ϕ, one would need to consider some general function of the dual kinetic term, $f(x)$, where $x = (\nabla \cdot (*A))^2$. Such nonlinear models are outside the scope of the present study, but in Table 1 we summarise briefly the relations between the equivalent formulation in more general cases than shown explicitly in here. On the other hand, if the field $(*A)$ has a more general potential than a mass term, the resulting reformulation naturally exhibits noncanonical kinetic terms. In the case $n = d-1$, it is thus a way to generate k-essence models [47] as pointed out by Gruzinov [48].

If $\xi \neq 0$ we obtain an unusual coupling between the generalised scalar field and curvature; previously such couplings have been applied in attempts to model a dynamical relaxation of the cosmological constant, dark matter and gravity assisted dark energy [49, 50, 51, 52].

$(*A)$	ϕ	$n = d-1$
Mass term	Canonical kinetic	Quintessence
General potential	Noncanonical kinetic	K-essence
Dual kinetic	Mass term	Chaotic inflation
Function	General potential	General scalar

Table 1: The correspondence between an original dual $(d-n)$ form $(*A)$ and the $(d-n-1)$ form field ϕ in the reformulation of the theory. The canonic dual with a mass can be rewritten as a massive gradient kinetic term, but more general Lagrangians are obtainable. In the last column we have indicated the type of a corresponding scalar field. The potential of the quintessence is given by the form of the kinetic term of the field A: in case of a canonical kinetic term of A, the quintessence model has a canonical mass term. This is also what we mean by the last line: a given function of the dual kinetic term , turns into a given potential function of the quintessence field.

2.3 Gauge invariance

The potential terms manifestly break gauge invariance. By performing the Stückelberg trick, we can promote the action into a gauge invariant form. Then we introduce a new field patterned after the gauge symmetry, whose transformation compensates for the gauge

symmetry violation. Only the antisymmetric part of the field counts, so in essence we will have an $(n-1)$-form as the following:

$$A = B + \frac{1}{m}\left[\partial\Sigma\right], \tag{2.21}$$

where $m > 0$ is a suitable mass scale. The new Lagrangian density,

$$\mathcal{L} = -\frac{1}{2(n+1)!}F^2(B) - \frac{1}{2}M^2\left(B + \frac{1}{m}\left[\partial\Sigma\right]\right)^2, \tag{2.22}$$

where M^2 is given by the effective mass including the contribution from the coupling, then has the gauge symmetry

$$\Sigma \to \Sigma + \Delta, \qquad\qquad B \to B - \frac{1}{m}\left[\partial\Delta\right], \tag{2.23}$$

where Δ is an arbitrary tensor with $(n-1)$ indices. In the gauge $\Sigma = 0$ we recover our original Lagrangian. We see that the field Σ has a kinetic term, and also a cross term. If we choose the specific gauge where the gradient of the Σ is orthogonal to our n-form[1] and set $m = \sqrt{|M^2|}$, the Lagrangian density becomes

$$\mathcal{L} = -\frac{1}{2(n+1)!}F^2(B) - \frac{1}{2}M^2B^2 - \frac{1}{2}\mathrm{sgn}(M^2)\left[\partial\Sigma\right]^2. \tag{2.24}$$

Now the Σ field, or more precisely the $(n-1)$-form one gets by anti-symmetrising the Σ, is a ghost if M^2 is negative. We have reached the conclusion that if the effective mass of an n-form is negative, there is an $(n-1)$-form ghost degree of freedom in the theory.

3. Homogeneous Cosmology

In this part we consider the use of n-forms for early cosmology and we thus restrict to the case $d = 4$ and to homogeneous cosmologies (that is the fields depend only on time). We consider the most simple metric compatible with anisotropy when required, namely an axisymmetric Bianchi I metric, which can be written as

$$ds^2 = -dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2 + dz^2\right)\right]. \tag{3.1}$$

We also define $H = \dot{\alpha}$ and we will refer to $\dot{\sigma}$ as the shear. The reason we restrict ourselves to axisymmetry is that in a general Bianchi I spacetime the stress tensor (2.7) is compatible with only one nonzero component of any n-form, and this results in an axisymmetric stress energy tensor. A caveat is that one may allow, in some cases, more components if they satisfy certain first order constraint equations. Fields obeying such constrains would not be dynamical in the usual sense, and typically exhibit decaying anisotropic stress. We do not consider these special cases in more detail.

[1] Alternatively one could reach our result by considering the limit $M^2 \to 0$. This limit is smooth and preserves the degrees of freedom, and the amount, though not the form of the gauge symmetry.

3.1 Zero-form

A zero-form is a scalar field and its application to inflationary cosmology has been intensively studied in the literature over the three past decades [53, 54, 55, 56]. Since a scalar field has a vanishing contribution to the anisotropic stress tensor, the anisotropy is decaying ($\dot{\sigma} \sim 1/a^3$) and one often study the cosmologies with scalar fields directly in a homogeneous and isotropic universe, that is with FRW symmetries.

3.2 One-form

Only one nonzero component is allowed by the Bianchi I symmetry, having two components would introduce nondiagonal components in the stress tensor (2.7). A zero component would be nondynamical, its equation of motion simply stating it vanishes or that the $V' \sim R/2$.

So let's consider spatial component

$$A_M = e^{\alpha(t)-2\sigma(t)}X(t)\delta_{Mx}.$$

Then $A^2(t) = X^2(t)$. The equation of motion (2.8) becomes

$$\ddot{X} + 3H\dot{X} + \left[2V' + (1+6\xi)(\dot{H}+2H^2) - 2H\dot{\sigma} - 2\ddot{\sigma} - (4-6\xi)\dot{\sigma}^2\right] X = 0, \tag{3.2}$$

Now the coupling $\xi = -1/6$ eliminates the effective mass terms. However, the model is unstable, as we see in subsection 4.2 and was first discussed in detail in [59]. Since the effective mass squared is negative, there is a scalar ghost, in accord with our generalised argument in subsection 2.3.

Now if we want to understand if this slow-roll dynamics is compatible with inflation with small anisotropy we need ρ, P and Π where the last quantity is defined from the anisotropic stress tensor π^i_j =diag$(-2\Pi, \Pi, \Pi)$. We specialise here to $V(x) = \frac{1}{2}m^2x$. By the usual definitions, the energy density, isotropic and anisotropic pressure come out as

$$\rho = \frac{1}{2}\left[m^2X^2 + (\dot{X}+HX-2\dot{\sigma}X)^2\right] + \xi\left[6HX\dot{X} + 3(H^2-\dot{\sigma}^2)X^2\right], \tag{3.3}$$

$$\begin{aligned} P = & \left[\frac{1}{6}(\dot{X}+HX-2\dot{\sigma}X)^2 - \frac{1}{6}m^2X^2\right] \\ & -2\xi\left[\dot{X}^2 - HX\dot{X} - X^2\left(m^2 + (1+6\xi)\dot{H} + H^2\left(\frac{5}{2}+12\xi\right) - 2H\dot{\sigma} - 2\ddot{\sigma} - \left(\frac{9}{2}-6\xi\right)\dot{\sigma}^2\right)\right], \end{aligned} \tag{3.4}$$

$$\Pi = \frac{1}{3}\left[(\dot{X}+HX-2\dot{\sigma}X)^2 - m^2X^2\right] - \xi\left[2\dot{H} + 4H^2 + 2\dot{\sigma}^2 - (\ddot{\sigma}+3H\dot{\sigma})\right]X^2 + \xi 2\dot{\sigma}X\dot{X}. \tag{3.5}$$

The general condition for the violation of the strong energy condition (and thus acceleration) is $\rho + 3P < 0$. In the minimally coupled case ($\xi = 0$), we see directly that

$$\rho + 3P = (\dot{X}+HX-2\dot{\sigma}X)^2 > 0, \tag{3.6}$$

and there is no possible accelerated expansion, that is no inflation.

However in the case $\xi = -1/6$ which is required to ensure slow-roll of the field

$$\rho + 3P = 2\dot{X}^2 - 4\dot{\sigma} X \dot{X} + X^2(10\dot{\sigma}^2 - m^2 + 2\ddot{\sigma} - 2\dot{\sigma} H) \simeq \left(2\ddot{\sigma} - m^2\right) X^2 \,, \tag{3.7}$$

from which the equation for $\ddot{a}/a$ can be deduced thanks to

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2(\rho + 3P)}{6} - 2\dot{\sigma}^2 \,. \tag{3.8}$$

If we can neglect $2\ddot{\sigma}$ in front of m^2 and assume a negligible shear $\dot{\sigma} \ll H$, then this leads apparently to exponential inflation $[a \sim \exp(Ht) = \exp(m\kappa X t/\sqrt{6})]$. However, it turns out that we can define an effective anisotropic stress is given by

$$\tilde{\Pi} \equiv \Pi + \frac{1}{6}(\ddot{\sigma} + 3H\dot{\sigma})X^2 = \frac{1}{3}\left[\dot{X}^2 + X\dot{X}(2H - 5\dot{\sigma}) + X^2(-4\dot{\sigma}H + 5\dot{\sigma}^2 - m^2 + \dot{H} + 3H^2)\right] \,,$$

in such a way that the evolution equation for the shear can be written as

$$\left(1 + \frac{\kappa^2}{6}X^2\right)(\ddot{\sigma} + 3H\dot{\sigma}) = \kappa^2 \tilde{\Pi} \,. \tag{3.9}$$

The dominant term on the right hand side of this equation is $\kappa^2 H^2 X^2$ and tends to increase the shear. For a field large enough ($\kappa^2 X^2/6 \gg 1$) which is anyway required to have enough e-folds of accelerated expansion, first $\ddot{\sigma} \simeq 6H^2$ with $\dot{\sigma} \simeq 0$, and $2\ddot{\sigma} > m^2$ from which we obtain $\rho + 3P < 0$. Additionally, after a transition period the shear is reaching $\dot{\sigma} \simeq 2H$ with $\ddot{\sigma} \simeq 0$. We deduce that these two conclusions contradict exponential inflation and negligible shear. Thus there is no inflation, that is accelerated expansion, with just one slow-rolling vector field.

3.3 Triads

Consequently, we need to assume that there are not one but several vector fields. The simplest model assumes a triad of vector fields [12], to ensure that there is no shear ($\dot{\sigma} = 0$). If we do so, we can assess quickly the fine-tuning involved. We can parameterise the difference of one vector field with respect to the two other vector fields by

$$X_1^2 = (1 + \epsilon)X^2/3, \quad X_2^2 = X_3^2 \equiv X^2/3 \tag{3.10}$$

with $\epsilon \ll 1$. Then if the fields are slow-rolling, if there is no shear to start with ($\dot{X}_i \ll HX_i$ and $\dot{\sigma} \ll H$) and if the field is rather large in order to ensure long enough inflation ($\kappa^2 X^2/6 \gg 1$), we obtain as long as the shear stays negligible

$$H^2 \simeq \frac{\kappa^2}{6} m^2 X^2 \,, \tag{3.11}$$

$$\rho + 3P \simeq X^2 \left[-m^2 + 2\epsilon\ddot{\sigma}\right] \,, \tag{3.12}$$

$$\tilde{\Pi} \simeq \epsilon X^2 H^2, \qquad \ddot{\sigma} \simeq 6\epsilon H^2 \,. \tag{3.13}$$

In order to have accelerated expansion, we thus need

$$\epsilon < m^2/(2\ddot{\sigma})\,, \tag{3.14}$$

which leads to

$$\epsilon < (\sqrt{2}\kappa X)^{-1}. \tag{3.15}$$

Since the number of e-folds is approximately given by

$$N \simeq \left(\frac{\kappa X}{2}\right)^2 - \frac{1}{2}\,, \tag{3.16}$$

then in order to obtain the $N \geq 70$ e-folds required to solve the problems of the standard hot big-bang model, we need $\kappa X > 17$ and thus the condition on ϵ is

$$\epsilon < 0.04\ . \tag{3.17}$$

The model with a triad of vector fields is thus fine-tuned but the fine-tuning is only going like $1/\sqrt{N}$ and can be considered as reasonable.

3.4 Two-form

If we take

$$A_{MN} = e^{\alpha(t)-2\sigma(t)}X(t)(\delta_{M0}\delta_{Nx} - \delta_{Mx}\delta_{N0}), \tag{3.18}$$

then $A^2(t) = -2X^2(t)$. However, now the kinetic term is identically zero, and the equation of motion (2.8) dictates an algebraic constraint for the field $X(t)(4V' + \xi R) = 0$, analogous to the vector case but now living in an anisotropic background. Such constrained models could be applied to construct effectively nondynamical cosmological fields like in the so called Cuscuton models or in modified gravity within the Palatini approach [60, 61, 62, 63]. Now however, the underlying theory does have more degrees of freedom. Here the fields are restricted only due to the homogeneity of the FRW or Bianchi I background, and therefore the perturbations about this background can propagate. Thus in principle they could also be responsible for the primordial spectrum of perturbations and structure in the universe. Furthermore, this means also that these models are not trivialised at large scales by averaging, which might occur for gravity modifications of the Palatini type [64, 65]. The formal reason for the time-like fields being algebraically constrained is that that the Bianchi I symmetry allows only time derivatives, but the zero index acting in the kinetic term must vanish in its antisymmetrisation if it appears in a component of the field.

Here we however consider a space-like two-form. Therefore let us take

$$A_{MN} = e^{2\alpha(t)+2\sigma(t)}X(t)(\delta_{My}\delta_{Nz} - \delta_{Mz}\delta_{Ny}). \tag{3.19}$$

Then $A^2(t) = 2X^2(t)$. The constraint (2.9) is identically satisfied by this ansatz. The equation of motion (2.8) yields

$$\begin{aligned}\ddot{X} \ + \ 3H\dot{X} + 2\Big[&2V'(A^2) + (1+3\xi)\dot{H} \\ &+ \ (1+6\xi)H^2 - \dot{\sigma}H + \ddot{\sigma} - (2-3\xi)\dot{\sigma}^2\Big]\,X = 0.\end{aligned} \tag{3.20}$$

The coupling $\xi = -1/6$ allows now to eliminate the effective mass due to H^2. Then a slow-roll suppressed mass term remains due to $\dot{H}$, plus the shear-terms. These are small and don't (necessarily) spoil slow-roll. However, since the effective mass squared is negative, there is a vector ghost, which follows directly from our generalised argument in subsection 2.3. This can also be confirmed explicitly by considering the perturbations in Minkowski space, see subsection 4.3.

Again defining the anisotropic stress as π^i_j =diag$(-2\Pi, \Pi, \Pi)$, we find now that

$$\begin{aligned}\Pi = &+6H^2 + 4\dot{H} - 2V + 4V'X^2 + 6\sigma^2 + \ddot{\sigma} + 3H\sigma \\ &+ \xi\left(-4\ddot{X}X - 4\dot{X}^2 + 2X^2\dot{H} + 3H^2X^2 - X^2\ddot{\sigma} + \dot{X}X\dot{\sigma} - 8\dot{X}XH - 3X^2\dot{H}\dot{\sigma}\right).\end{aligned} \tag{3.21}$$

This is similar to with the vector expression (3.5) of the vector case. Thus, following similar arguments as there, one could deduce that a single two-form cannot support an inflating background for many e-folds, since the anisotropy of the solution tends to grow quickly. Further, one could again add a triad of two-forms to ensure an isotropic background. These three two-forms would again have to be tuned to be equal with the (qualitatively) same accuracy as in the vector case. The similarity of the vector and two-form cases can be traced to the duality discussed in general terms in section 2.2. In the next section we apply it in more detail in the specific cases of vector $\leftarrow$ two-form and two-form $\leftarrow$ vector (these cases are not equal due to the nonminimal coupling).

3.5 Three-form

If we would consider the ansatz $A = e^{2\alpha+\sigma}X(t)dt\wedge dy\wedge dz$, we would again have an algebraic model, $X(12V' - \xi R) = 0$. Thus, like in the previous cases, we consider only spatial indices. Since there are three of them, no direction is picked up, and we can restrict to the case of FRW, $\sigma(t) = 0$. We write $A = e^{3\alpha(t)}X(t)dx\wedge dy\wedge dz$. It follows that $A^2(t) = 6X^2(t)$. The Friedmann equation we obtain is

$$3\left[\frac{1}{\kappa^2} - \left(\frac{3}{2} + \xi\right)X^2\right]H^2 = \frac{1}{2}\dot{X}^2 + 3(1+2\xi)H\dot{X}X + \tilde{V}(X^2) \tag{3.22}$$

and the equation of motion for the field is

$$\ddot{X} + 3H\dot{X} + 3\left[\frac{2}{3}\tilde{V}'(X^2) + 4\xi H^2 + (1+2\xi)\dot{H}\right]X = 0\,, \tag{3.23}$$

where $\tilde{V}(X^2) \equiv V(A^2) = V(6X^2)$. Thus, the nonminimal coupling introduces an extra mass term, just like for a scalar field. We set $\xi = 0$ in the following. Thus we consider a different case from Refs.[31, 32], where suitably fixed nonminimal gravity couplings were used to turn the equation of motion (3.23) into the Klein-Gordon form. Now the effective energy density and pressure may be then written as

$$\rho_X = \frac{1}{2}\left(\dot{X} + 3HX\right)^2 + \tilde{V}(X^2), \tag{3.24}$$

$$P_X = -\frac{1}{2}\left(\dot{X} + 3HX\right)^2 + 2\tilde{V}'(X^2)X^2 - \tilde{V}(X^2). \tag{3.25}$$

With quadratic potential the behaviour is the *reverse* of that of a scalar field with Hubble friction contributing to the kinetic energy: the kinetic piece gives a negative, the potential a positive contribution to the pressure. If V' can be neglected for some general potential though, both contributions are negative. Then the equation of state mimics a cosmological constant though the field could evolve. The condition for inflation is that

$$\frac{6\ddot{a}}{\kappa^2 a} = \left(\dot{X} + 3HX\right)^2 + 2\tilde{V}(X^2) - 6\tilde{V}'(X^2)X^2 > 0. \tag{3.26}$$

Thus, we do not need slow roll to get inflation. A three-form seems to accelerate a FRW universe more naturally than a scalar field. To realise phantom inflation in this model one needs a negative slope for the potential,

$$\dot{H} = -\kappa^2 X^2 \tilde{V}'(X^2) > 0. \tag{3.27}$$

Now the model seems stable, since the Stückelberg argument does not give a ghost when $V > 0$, and since we have just a canonical action without the dangerous nonminimal couplings. The quantitative predictions of this class of models will be considered elsewhere.

3.6 Four-form

The kinetic term for a four-form is trivial in four dimensions. The fact that such term can lead to a constant contribution to energy density has been employed in an anthropic solution to the cosmological constant problem [66]. With general potential however, the field can have nontrivial contribution. The variation of the action with respect to the field $A^2 = \varphi$ leads to an algebraic constraint for the field, $48V'(\varphi) = \xi R$. If the solution is plugged back into the action, we recover a higher order gravity theory in the form of a metric $f(R)$ theory

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(\frac{1}{\kappa^2} - \xi\varphi(R)\right)R - V(\varphi(R))\right]. \tag{3.28}$$

When $\xi = 0$ this reduces to general relativity with a cosmological constant given by the minimum of V. If V is a mass term for the potential A, only constant-Ricci solutions are compatible with such a theory. Thus our world is not compatible with a simple $V(x) \sim x$ with coupling to the curvature.

One more interesting simple example is a self-interaction which may be seen as a mass term for the field squared, $V(x) \sim x^2$. This results in curvature-squared R^2 correction to the Einstein-Hilbert action. Furthermore, one notes that there exists also a trivial solution to the equation of motion, $A = 0$. Thus there are two branches of, GR and $f(R)$. Perhaps the other branch could be used to "switch off" the $f(R)$ theory like in some kind of phase transition when A crosses zero. On the other hand, to exclude the GR solutions, one could add a $1/A^2$ term in the potential.

4. Perturbations

4.1 0-form

This is a scalar field, and the general Bianchi I case (not necessarily axisymmetric) perturbation theory is built in [67] and slow roll inflation is studied in [68]. The axisymmetric

n	# dof	Π	ξ	Sec.	Comment	Ref.
0	1	0	0	3.1	A scalar field	[73]
1	4	$\sim X^2$	$-\frac{1}{6}$	3.2	Needs nonminimal coupling for SR	[12]
2	6	$\sim X^2$	$-\frac{1}{6}$	3.4	Needs nonminimal coupling for SR	-
3	4	0	0	3.5	isotropic SR inflation	-
4	1	0	$\neq 0$	3.6	metric $f(R)$ gravity	[74]

Table 2: A summary. For each class of forms, we indicate 1) the number of degrees of freedom (in the cosmological background, the effective number could of course be less) 2) the anisotropic stress of the model (which is zero for isotropic cases) 3) the section in this paper where we focus on the case 4) a comment about the general nature of model 5) a reference for some related earlier study.

Bianchi I case is also studied in [69]. These two studies have shown that, though the perturbation theory is well-defined for a scalar field in an anisotropic background, the quantisation of the canonical degrees of freedom is lacking from a well-defined asymptotic adiabatic vacuum. Indeed the fact that a scalar field has no anisotropic stress implies that the shear is decreasing, which means that it necessarily increases when going back in time. As a result the perturbation theory fails to be predictive above a certain scale which corresponds to the modes which have exited the Hubble radius when the anisotropy was still largely dominating. However for such models, the power spectrum converges toward a nearly scale-invariant shape for scales smaller than this limiting scale, and if inflation has lasted long enough the larger modes can still be outside the Hubble horizon in the present universe and we recover the usual predictions obtained when the background homogeneous cosmology is taken with the FRW symmetries.

4.2 1-form

Performing the full perturbation theory can be very tedious. Here we only try to find the main behaviour of the result by simplifying the perturbation scheme. We thus ignore the backreactions and ignore the perturbations in the metric and work on a Minkowski background. Since the gauge invariant variables involve a combination of the perturbations of the field and of the metric, this is not a bad approximation. This boils down to consider a test field with a mass directly related to the nonminimal coupling. This is similar to what is undertaken in [59]. (About nongaussianity in of perturbations in the presence of vector fields, see [70, 71, 72]).

We start from the action (2.4) with $V(x) = \frac{1}{2}M^2x^2$ and a Minkowski background, and (see [59]) setting $A_M = (\alpha_0, \partial_i\alpha + \alpha_i)$ we obtain

$$S = \int d\eta d^3k \left\{ \frac{1}{2}\left[|\alpha_i'|^2 - \left(k^2 + M^2\right)|\alpha_i|^2\right] + \frac{1}{2}\left[k^2|\alpha'|^2 \right.\right.$$
$$\left.\left. - \; k^2(\alpha'^*\alpha_0 + cc) - M^2k^2|\alpha|^2 + \left(k^2 + M^2\right)|\alpha_0|^2\right]\right\}$$

we obtain a constraint from α_0 and plugging it back the action is recast as

$$S = \int d\eta d^3k \frac{k^2 M^2}{2} \left[\frac{|\alpha'|^2}{k^2 + M^2} - |\alpha|^2 \right] + \int d\eta d^3k \frac{1}{2} \left[|\alpha_i'|^2 - \left(k^2 + M^2\right) |\alpha_i|^2 \right] \qquad (4.1)$$

We conclude that if $M^2 < 0$ the mode α is not well behaved but it is if $M^2 > 0$. See however the discussion 4.5.

One more way to look at the possible appearance of the ghost mode is to employ the dual description of the section 2.2. The non-minimally coupled vector Lagrangian is dual to a massive two-form B with a noncanonical kinetic term. The Lagrangian for this two-form B reads explicitly

$$\mathcal{L} = -\frac{3}{\xi R/m^2 + 1} F^2(B) - \frac{1}{12} m^2 B^2. \qquad (4.2)$$

Thus, since now the prefactor of the kinetic term becomes negative for $\xi < 0$ and $m^2 < R$ it is a ghost, as least as long as R may be regarded as a constant background field. This also clarifies the similarities between vector field and two-form inflation. The vector inflation *is* the two-form inflation, where one has a bare mass term but a ghost (or in general, more complicated) kinetic term.

4.3 2-form

We start from the action (2.4) in Minkowski background and with $V(x) = \frac{1}{4}x^2$. We then decompose A_{MN} in the following manner

$$A_{0i} = \partial_i E + E_i \qquad\qquad A_{ij} = \epsilon_{ijk}(\partial^k B + B^k) \qquad (4.3)$$

with $\partial_i E^i = \partial_i B^i = 0$. Now the perturbed action is, up to total derivatives

$$S = \int d\eta d^3k \left[\frac{1}{2} B_i' B^{i\prime} + \frac{1}{2}\partial_i B' \partial^i B' - \frac{1}{2}\Delta B \Delta B + \frac{1}{2}\partial_i E_j \partial^i E^j - B^{i\prime} \epsilon_{ijk} \partial^k E^j \right] \qquad (4.4)$$

$$+ \int d^4x \left[-\frac{1}{2} M^2 B_i B^i + \frac{1}{2} M^2 E_i E^i - \frac{1}{2} M^2 \partial_i B \partial^i B + \frac{1}{2} M^2 \partial_i E \partial^i E \right] \qquad (4.5)$$

We can go in Fourier space and decompose the vector terms on an orthonormal basis $e_1^i, e_2^i, \hat{k}^i$

$$B^i = \sum_{a=1,2} \mathrm{i} B^a e_a^i. \qquad (4.6)$$

As constraints, in Fourier space, we obtain

$$E = 0 \qquad (4.7)$$

$$(M^2 + k^2) E_a, = \mathcal{M}_a^{\ b} k B_b' \qquad (4.8)$$

where

$$\mathcal{M}_a^{\ b} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \qquad (4.9)$$

So E and E_i are constrained and once replaced in the action we obtain

$$S = \int d\eta d^3k \left\{ \frac{k^2}{2} \left[|B'|^2 - \left(k^2 + M^2\right) |B|^2 \right] + \frac{M^2}{2} \left[\frac{|B_i{}'|^2}{k^2 + M^2} - |B_i|^2 \right] \right\} . \qquad (4.10)$$

So now the well behaved part is B and the part which is not well behaved when $M^2 < 0$ is B_i but the argument is similar. See the discussion in the next section about it.

Again, like in the vector case, we can exploit at the dual description derived the section 2.2 to recast the two-form into a vector. We already know that the non-minimal coupling will become a kinetic coupling and that the canonical kinetic term transforms into canonical mass term. Writing then explicitly the Lagrangian for the vector B which is equivalent to the two-form reads,

$$\mathcal{L} = -\frac{1}{\xi R/m^2 + 1} F^2(B) - \frac{1}{2} m^2 B^2. \qquad (4.11)$$

In the two-form decription, one has to perform the decomposition or use the Stückelberg trick to fish out the vector degree of freedom that appears then with the wrong sign. Alternatively however, one may use the dual description as a vector field and see immediately when the ghost seems to appear. Now, of course again the prefactor of the kinetic term becomes negative for $\xi < 0$ and $m^2 < R$.

4.4 Three-form

The arguments of the section 2.2 and of the section 2.3 both support the stability of the three-form, whereas in the vector and two-form cases both seemed to have problematical implications. More specifically, the gauge symmetry completion of the three-form with positive mass is a canonical two-form. Also, the dual scalar field is not a ghost unless the mass was negative. Also, the three-form cosmology can do without anisotropy and nonminimal couplings, which may otherwise introduce problems with stability. This strongly supports the conclusion that three-forms models *can* be viable. Investigation of more specific models will be presented elsewhere [57, 58].

4.5 About the nature of instability

From our analysis in subsection (2.3), it is clear that in flat spacetime with constant $M^2 < 0$ there is a ghost. However, it is more subtle to see how this analogy can be used to analyse a nonminimal coupling in FRW spacetime which is not Ricci-flat. An argument for not using this analogy might seem to be that the gravitational degrees of freedom in R have to integrated out in order to write the quadratic action in a canonical form. The full computation which consists in perturbing quadratically both the metric and the field has been performed in Ref [59], though for the specific model of Ref [75] (see also ([76]). What is found is that the coefficient in front of the kinetic term changes its sign. When it does so, the solution could be expected to diverge. The appearance of ghost seems to be linked with a classical divergence. This is understandable, since if there is no consistent classical solution, there is no possibility to quantise. Similar relation between classical singular behaviour and an appearance of a quantum ghost has been found in the Gauss-Bonnet cosmology [78]. There also a divergence of cosmological perturbations (which was first

noticed in the context of first-loop string cosmology [79]) is directly linked with the change of the sign of a kinetic term of an effective degree of freedom [80, 81]. In the case of higher inverse derivative gravity [82], the cosmological background reaches a sudden singularity at the moment that a sign flips in front of a scalar degree of freedom of the theory [83, 84]. Again a classical divergence reflects a fundamental quantum problem.

However, the divergence of the classical solution which was expected to appear around horizon crossing has been claimed to in fact, not be there in the solutions which were presented explicitly in Ref. [72] where the longitudinal mode was also analysed in detail. It was also argued that the ghost might not be dangerous to the theory if it appears at a time when all the couplings to other field are negligibly small. No divergence was directly seen in Ref [77]. The question then arises whether the apparent classical and quantum problems both disappear at more careful scrutiny. Finally, we remind that though the above condiserations are made for a vector field, by the strong analogy we have developed during this paper, they apply (almost) as such to the two-form as well.

5. Conclusions

In this work we have investigated anisotropic and isotropic slow-roll inflation supported by n-forms. We have used both canonical field strengths and their duals, allowing them to have a potential and a nonminimal coupling to curvature if necessary. New anisotropic solutions were found for two-form, generalising vector inflation. However, we have found that, as in the case of vector inflation, inflation driven by the two-form would be unstable. One type of problem is that a new field appears into the model which is pathological when the effective mass squared is negative. We showed this also by considering the action for perturbations in the simple flat case. In addition, the stability of a background with initially small shear was investigated, which for one-form field seems critical but with a triad more reasonable.

New viable isotropic solutions were found. Three-forms could naturally support inflation. In particular, the three-form could inflate even if the field wasn't slowly rolling and could realise a phantom inflaton depending on the slope of the potential. Moreover, we have also shown that some n-form actions are equivalent to $f(R)$ gravity and some to scalar field models with a possible nonminimal coupling. These observations seem to strongly motivate further investigation of form-driven inflation. In particular, one is interested in the observational predictions for the nature of primordial perturbations and their non-Gaussianity.

More compactly, the two main lessons of our study are the following.

- The assumption that inflaton is driven by a scalar field is not robust. We have found simple and viable inflationary models driven by a form field.

- The assumption that the inflaton appears isotropic at large scales seems justified. The anisotropic solutions in the simplest cases seem to generically require nonminimal couplings and feature instabilities.

The findings in each of the five cases are summarised in Table 2. On the more formal side, a reformulation of the dual forms with nonminimal coupling was proposed which seems to generalise some scalar-tensor models in a way which could be possibly useful in the context of alternative dark matter theories and the cosmological constant problem. Let us remark that though the inflaton seems isotropic, anisotropy could originate from fields playing role as impurity during inflaton or curvaton generating the perturbations [85, 86, 87]. In particular, gauge invariance preserving form field could naturally leave percent level anisotropic hair if nonminimally coupled with the inflaton [88].

It would be interesting to apply these considerations during the dark energy era. Then it becomes relevant to study the dynamics of anisotropic component in the presence of matter fluids [89, 90] and its effects to the formation of large scale structure [91, 92]. Indeed, possible anisotropic effects in the CMB might be due to an unexpected property of the late acceleration, they do not have to be imprinted already in the primordial inflationary spectrum of fluctuations. Furthermore, interesting constraints could be potentially obtained from possible CMB B-modes polarisation within these anisotropic cosmologies. Such possibility is enhanced from the transformation of E-modes to B-modes due to the shear [93].

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